

Tracking Performance of Bitcoin and Gold Leveraged ETFs

Hongfei Tang, Ph.D., CFA
Professor of Finance
Stillman School of Business
Seton Hall University
400 South Orange Avenue
South Orange, NJ 07079, USA
Phone: (973) 761-9151; Email: hongfei.tang@shu.edu

Xiaoqing Eleanor Xu, Ph.D., CFA^{*}
Professor of Finance
Stillman School of Business
Seton Hall University
400 South Orange Avenue
South Orange, NJ 07079, USA
Phone: (973) 761-9209; Email: xuxe@shu.edu

June 2026

^{*} Corresponding author. Xu gratefully acknowledges financial support from the Institute for International Business at Seton Hall University.

Tracking Performance of Bitcoin and Gold Leveraged ETFs

Abstract

This study investigates the tracking performance of Bitcoin leveraged exchange-traded funds (ETFs) relative to Gold ETFs, providing the first systematic analysis of cryptocurrency futures-based ETFs and those linked to a mature commodity futures market. Using daily data on funds with leverage ratios of $1\times$, $2\times$, $-1\times$, and $-2\times$, we examine performance over 1-, 5-, and 20-day holding periods and decompose total return deviations into compounding, NAV, and inefficiency components. The results show that compounding deviation is the dominant source of multi-day tracking error, with its impact amplified by Bitcoin's volatility. While Gold ETFs exhibit NAV deviations broadly consistent with cost-of-carry predictions, Bitcoin ETFs show no such pattern, reflecting investor sentiment, liquidity frictions, and unregulated trading. Inefficiency deviations are transitory and economically minor in both markets but larger for Bitcoin ETFs. Overall, the findings highlight structural differences between cryptocurrency and traditional commodity ETFs and carry important implications for investors and policymakers.

JEL Classification: G11, G12, G14

Keywords: Leveraged Exchange-Traded Funds; Tracking performance; Cryptocurrency; Compounding Deviation; Bitcoin and Gold Futures

Tracking Performance of Bitcoin and Gold Leveraged ETFs

1. Introduction

Since the debut of Bitcoin in 2009, cryptocurrencies empowered by blockchain technology have evolved into a distinctive and rapidly growing asset class, attracting retail and institutional investors seeking portfolio diversification, high-return potential, and exposure to the emerging digital finance landscape. The introduction of futures-based cryptocurrency exchange-traded funds (ETFs) in October 2021 enabled U.S. investors to access cryptocurrencies such as Bitcoin and Ethereum through liquid, accessible, and regulated financial instruments. Tang, Xie, and Xu (2025) demonstrate that these cryptocurrency ETFs have provided high pricing efficiency and strong tracking performance to cryptocurrencies.

Since June 2022, a suite of leveraged ETFs (LETFs) has been launched in the U.S., offering investors ultra (2x), inverse (-1x), and ultra inverse (-2x) daily exposure to Bitcoin and Ethereum. Given the extreme volatility and unique market dynamics of the underlying cryptocurrencies, important questions emerge regarding their tracking performance and the potential long-term value erosion of these daily leveraged instruments over various holding periods. As with traditional LETFs, these funds are subject to compounding effects, roll costs, market frictions, and tracking deviations arising from their futures-based structures, but the magnitude and persistence of such deviations in digital asset markets remain largely unexplored.

The primary objective of this study is to conduct a rigorous empirical examination of the tracking performance of Bitcoin LETFs, with Gold LETFs serving as a mature reference point within the class of commodity futures-based LETFs. An additional motivation for comparing

Bitcoin LETFs with Gold LETFs stems from the perception of Bitcoin as “digital gold.” Because Bitcoin has a fixed supply (capped at 21 million coins) and is often viewed as a potential store-of-value asset, it is frequently compared with gold in both the academic literature (Gkillas and Longin, 2019) and professional outlets such as *The Wall Street Journal* (February 19, 2021), *Reuters* (November 12, 2024), and *Barron’s* (December 7, 2024). Comparing Bitcoin LETFs with Gold LETFs therefore allows us to assess whether cryptocurrency-based leveraged products exhibit performance dynamics similar to those of established commodity-linked LETFs.

Specifically, we address four key questions: (i) How accurately do Bitcoin LETFs track their daily target multiples? (ii) What are the primary sources of tracking error over multiple-day horizons? (iii) How do market factors -- such as volatility, liquidity, and fund flows -- affect these deviations? and (iv) How does the tracking behavior of cryptocurrency LETFs compare with that of Gold LETFs, which represent a more established and stable asset class?

Our empirical results reveal several important findings. First, Bitcoin LETFs track their daily target multiples closely, with insignificant beta deviations, similar to Gold LETFs. Second, compounding deviation emerges as the dominant source of multi-day tracking error, and its magnitude is substantially amplified by the extreme volatility of Bitcoin. Third, while Gold LETFs exhibit NAV deviations broadly consistent with the cost-of-carry framework, Bitcoin LETFs display less systematic patterns, reflecting the influence of investor sentiment, liquidity frictions, and less regulated markets. Overall, these results highlight important structural differences between cryptocurrency and Gold LETFs.

The remainder of the paper proceeds as follows. Section 2 presents the theoretical framework and methodology used to decompose LETF return deviations into compounding, NAV, and inefficiency components. Section 3 describes the data and provides summary statistics for Bitcoin

and Gold LETFs. Section 4 reports the empirical results, while Section 5 provides the conclusion and implications.

2. Theoretical Framework and Methodology

Building on the return deviation frameworks established by Tang and Xu (2013) for leveraged ETFs and extended by Tang and Xu (2016) to futures-based LETFs, this study conducts a systematic analysis of the tracking performance of cryptocurrency LETFs. These frameworks are particularly well suited to the study of LETFs, as they decompose tracking outcomes into well-defined components and enable rigorous comparison across asset classes.

We begin by calculating the LETF's actual total return based on market prices and its NAV return based on reported net asset value. We then compute two benchmarks: the target return, defined as the daily leveraged return on the underlying index compounded over the holding period, and the naïve expected return, defined as the daily leveraged multiple applied to the cumulative return of the underlying index. We decompose the LETF total return deviation over different holding periods (between actual total return and naïve expected return) into three key components: compounding deviation (difference between the target return and the naïve expected return), NAV deviation (difference between NAV return and target return), and inefficiency deviation (difference between the LETF actual return and the NAV return). This framework accounts for the compounding effect due to the daily leverage reset, fund operations and frictions in NAV calculation, as well as potential market inefficiency in price tracking relative to NAV.

Using daily data from the whole family of cryptocurrency LETFs, with leverage ratios of $1\times$, $2\times$, $-1\times$, and $-2\times$, over 1-day, 5-day, and 20-day holding periods, we quantify the magnitude and persistence of return deviations and identify the primary drivers of performance erosion. We

further investigate how market volatility, fund flows, and liquidity conditions influence the magnitude of each deviation component, offering a more dynamic understanding of the behavior of these instruments. We also compare the tracking performance of cryptocurrency ETFs with those of Gold ETFs, a family of futures-based commodity ETFs (Guo and Leung, 2016) that are of closest relevance to cryptocurrencies. This comparative analysis allows us to contextualize the performance of crypto ETFs within a broader class of futures-based ETFs, highlighting both the similarities and distinctive features of cryptocurrency-linked products.

Overall, this framework allows us to quantify the structural limitations, risk characteristics, and performance drivers of cryptocurrency ETFs, providing critical insights for investors, portfolio managers, and policymakers into the suitability, risk profile, and structural limitations of these novel financial instruments.

3. Data Collection and Summary Statistics of Daily Variables

Our data are primarily sourced from Bloomberg and cover the period from the Bitcoin ETF inception date of June 21, 2022 to December 31, 2024. Table 1 presents summary statistics for daily returns, tracking performance, and other relevant daily variables for Bitcoin and Gold funds, respectively. Specifically, the Table includes statistics for funds' actual total return, NAV return, target return, and the underlying index return, enabling an initial assessment of tracking performance. For example, the mean Total Return Deviation reflects the average difference between the fund's actual performance and its intended target. Additional variables—such as fund price, NAV, assets under management, fund flow, lagged fund valuation premium, turnover, and bid-ask spread—offer further context for understanding the funds' trading dynamics and liquidity. Definitions of all variables are provided in Appendix Table A1.

Appendix Table A2 lists the sample funds analyzed in this paper. As shown in the table, the first Bitcoin ETF, ProShares Short Bitcoin ETF (ticker: BITI), is an inverse ($-1\times$) Bitcoin fund that was launched on June 21, 2022. The second Bitcoin ETF is a double ($2\times$) Bitcoin fund with the ticker BITX, launched on June 27, 2023. As of December 2024, BITX is the largest ETF among all Bitcoin and Gold ETFs, with assets under management of \$3.1 billion. Since BITX is both larger and older than the later-launched $2\times$ Bitcoin fund BITU, we exclude BITU from our final sample.¹ The inverse double ($-2\times$) Bitcoin fund, SBIT, was launched on April 2, 2024. For each of these Bitcoin ETFs, the sample period begins on the first date for which data are available and ends on December 31, 2024.

For comparison purposes, we also include the unlevered ($1\times$) futures-based Bitcoin ETF (ticker: BITO), the unlevered ($1\times$) Gold ETF (ticker: GLD), the double ($2\times$) Gold fund (ticker: UGL), and the inverse double ($-2\times$) Gold fund (ticker: GLL).² To ensure consistency in the sample period, we restrict the data for these funds to match the sample period of the longest-running Bitcoin ETF, BITI.

Panel A of Table 1 presents daily summary statistics for Bitcoin funds. The mean daily total return for the ($1\times$) Bitcoin fund is 0.28%, with a standard deviation of 3.56%, indicating high volatility. For ($2\times$) Bitcoin funds, the mean daily total return increases to 0.58%, with a higher standard deviation of 6.68%, reflecting the amplified return and risk profile of leveraged exposure. In contrast, the ($-2\times$) Bitcoin fund shows a mean daily total return of -0.44% with a standard

¹ The tracking performance of the two ($2\times$) Bitcoin ETFs, BITX and BITU, is largely consistent. Results on BITU are available upon request.

² We initially considered the ($-1\times$) DB Gold Short ETN (ticker: DGZ) and the ($-2\times$) DB Gold Double Short ETN (ticker: DZZ) for inclusion in our study sample. However, because Exchange-traded Notes (ETNs) lack the creation/redemption mechanism, do not have available historical NAV data in Bloomberg, and are not directly comparable to inverse Bitcoin ETFs, we exclude them from our analysis.

deviation of 7.06%. The sample includes 636 daily observations for the (1×) and (-1×) funds, 381 for the (2×) fund, and 189 for the (-2×) fund.

As shown in Panel A of Table 1, the (1×) Bitcoin fund exhibits closely aligned average returns across total return, NAV return, target return, and the underlying index return, registering 0.28%, 0.28%, 0.29%, and 0.29%, respectively. The total return deviation, defined as the fund's total return minus its target return, has a mean of -0.01% and a standard deviation of 0.97%. The NAV return deviation, calculated as the fund's NAV return minus its target return, averages -0.02%, with a standard deviation of 0.96%. The inefficiency return deviation, defined as the difference between the fund's total return and its NAV return, has a mean of 0.00% and a standard deviation of 0.16%. These results suggest that, on average, the fund tracks its underlying index effectively, although the standard deviations indicate that daily deviations can be sizable.

As shown in Panel A of Table 1, for the (2×) Bitcoin fund, the average total return and NAV return are both 0.58%, while the target return is 0.69%. These figures result in an average total return deviation of -0.11%, an NAV return deviation of -0.11%, and an inefficiency return deviation of -0.01%. The average daily total return deviation for the (2×) Bitcoin fund is notably larger than that of the (1×) Bitcoin fund (-0.11% vs. -0.01%). This difference may be attributed to factors such as the higher fund management fee (1.9% vs. 0.95% annually, as shown in Appendix Table A2) and the cost of borrowing required to achieve leverage. We will examine these potential causes in more detail later.

As shown in Panel A of Table 1, for the (-2×) Bitcoin fund, the average total return and NAV return are both -0.44%, while the target return is -0.49%. This results in an average total return deviation of 0.05%, a NAV return deviation of 0.05%, and an inefficiency return deviation of 0.00%. The positive average total return and NAV return deviations are somewhat surprising

and may be attributed to interest income earned from shorting Bitcoin futures to achieve the inverse leverage. Similarly, for the $(-1\times)$ Bitcoin fund, the average total return and NAV return are both -0.26% , with a target return of -0.29% . This corresponds to average total and NAV return deviations of 0.03% , and an inefficiency return deviation of 0.00% .

As shown in Panel A of Table 1, the $(2\times)$ Bitcoin fund has the largest average daily assets under management (AUM) at \$1.073 billion, consistent with Appendix Table A2. All four Bitcoin funds exhibit positive average daily fund flows: 1.4% for the $(1\times)$ fund, 5.67% for the $(2\times)$ fund, 0.47% for the $(-2\times)$ fund, and 0.42% for the $(-1\times)$ fund, indicating growing investor interest. The average daily fund turnover rates are 16% for $(1\times)$, 18% for $(2\times)$, 109% for $(-2\times)$, and 54% for $(-1\times)$, reflecting short average holding periods, especially for the inverse funds. The rapid turnover of LETFs is expected, as they are designed to deliver daily target multiples rather than leveraged returns over longer horizons, which we will examine in detail later.

Panel B of Table 1 presents summary statistics for Gold ETFs for comparison purposes. The number of observations for all these funds is 636, matching the longest sample of the Bitcoin LETF in this study. For the unlevered $(1\times)$ Gold fund, the mean daily total return, NAV return, target return, and underlying index return are all 0.06% . This results in average deviations of zero for total return deviation, NAV return deviation, and inefficiency return deviation, demonstrating superior average tracking performance of the spot-based $(1\times)$ Gold fund. In comparison, the $(1\times)$ Bitcoin fund has a higher average daily NAV return (0.28% vs. 0.06%) and a larger standard deviation (3.56% vs. 0.86%).

The futures-based $(2\times)$ Gold fund has an average daily total return of 0.09% , NAV return of 0.09% , and target return of 0.12% . This results in average deviations of -0.03% for both total return and NAV return, and an average inefficiency return deviation of 0.00% . These deviations

are smaller in magnitude than those of the Bitcoin counterparts. Similarly, the standard deviations of total return deviation and NAV return deviation are also lower compared to the Bitcoin funds.

The (-2×) Gold fund has an average daily total return of -0.07%, NAV return of -0.07%, and target return of -0.12%. This results in average total return deviation and NAV return deviation of 0.05%, with an average inefficiency return deviation of 0.00%. These average deviations are consistent with those of the Bitcoin counterparts. However, the standard deviations of total return deviation and NAV return deviation are smaller than those of the Bitcoin funds.

In contrast to the positive average fund flows observed in Bitcoin funds, the three Gold funds exhibit negative average fund flows. Additionally, the fund turnover and bid-ask spreads for the Gold funds are lower than those of their Bitcoin counterparts.

4. Empirical Results

4.1. Daily Beta Estimation

Table 2 presents the daily beta estimation results using the single-index model to assess the exposure of Bitcoin and Gold funds to their respective underlying indexes. The market price beta is computed by regressing the actual total return of each fund on the return of the underlying index, while the NAV beta is computed by regressing the fund's NAV return on the index return. The beta deviation is calculated as the difference between the estimated fund beta and the fund's stated target multiple.

In Panel A, we regress the actual total return of each Bitcoin fund on the spot Bitcoin return to obtain the Bitcoin fund's market price beta. As shown, the beta of the (1×) Bitcoin fund is 1.02,

which is not statistically different from its target beta of 1. The intercept is -0.02%, or -2 basis points per trading day, which may partly reflect the fund's annual expense ratio of 0.95%. The R-squared is 92.6%.

For the (2×) Bitcoin fund, the estimated beta is 2.026, also not significantly different from the target multiple of 2. The intercept is -0.1189%, or -11.89 basis points per day, which may result from the higher expense ratio of 1.90% per year and the cost of leverage needed to achieve the 2× exposure. The beta of the (-2×) Bitcoin fund is -2.008, with an intercept of 0.0525%. This positive intercept may be attributed to interest income from shorting Bitcoin futures contracts. Similarly, the beta of the (-1×) Bitcoin fund is -1.0222, with an intercept of 0.038%. All constant terms for the Bitcoin ETFs are not statistically significantly different from zero.

Panel B reports the market price beta estimates for the Gold funds. In column (1), regressing the daily actual total return (based on fund market price) of the (1×) spot-based Gold fund on the spot gold return yields a market price beta of 0.9853, which is statistically different from its target multiple of 1 at the 10% level. Compared to the (1×) Bitcoin fund, the beta deviation for the (1×) Gold fund is smaller in magnitude; however, its smaller standard error results in greater statistical significance. The intercept for the (1×) Gold fund is -0.0006%, which is not statistically different from zero. The magnitude of this negative intercept is also smaller than that of the (1×) Bitcoin fund, potentially due to the Gold fund's lower annual expense ratio of 0.40% and its spot-based tracking approach (instead of BITO's futures-based tracking).

As shown in Panel B, column (2) of Table 2, the (2×) futures-based Gold fund has a beta of 1.9798, which is not statistically different from its target exposure of 2. The intercept is -0.0285%, likely reflecting the impact of the fund's expense ratio and the financing costs associated with leverage. Compared to column (2) of Panel A, this intercept is smaller in magnitude (-0.0285%

vs. -0.1189%). In column (3), the (-2×) Gold fund has a beta of -1.9727, which is statistically different from its target multiple at the 10% level. The intercept is 0.0482%, significant at the 1% level, potentially due to interest income earned from shorting gold futures. The constant terms for both Gold LETFs are statistically significant from zero.

Panel C presents the results of the market price beta deviation analysis for Bitcoin funds, obtained by regressing the deviation of actual total return from the target return on spot Bitcoin returns. As shown, none of the betas deviate significantly from their targets. Panel D reports the corresponding results for Gold funds. The beta deviations for the (1×) and (-2×) Gold funds are statistically significant at the 10% level.

Panel E presents the NAV return beta estimates for Bitcoin funds. Consistent with Panel A, none of the estimated betas differ significantly from their target values. Panel F shows the NAV return beta estimates for Gold funds, where all betas significantly deviate from their respective targets. These deviations may be attributed to non-synchronous timing between the NAV calculation and Bloomberg's reported daily closing price of the spot Gold market.

4.2. Daily Return Deviation Analysis

Because an LETF is designed to track its underlying index with a daily target multiple, it is essential to assess how effectively these funds achieve their daily performance objectives. The daily total return deviation—defined as the difference between an LETF's actual total return and its target return—can be decomposed into two components: (i) the NAV deviation, the difference between the NAV return and the target return, which arises from fund management tracking error, trading costs, and nonsynchronous trading issues; and (ii) the inefficiency deviation, the difference

between the actual total return and the NAV return, which reflects market frictions and inefficiencies.

4.2.1. NAV Return Deviation Analysis

Table 3 presents the results of the daily NAV return deviation analysis, utilizing OLS regressions with heteroskedasticity-robust standard errors. This analysis explores the factors influencing the deviation of the fund's Net Asset Value (NAV) return from its target return. All variables in these regressions are winsorized at the 1% level to mitigate the impact of outliers.

As shown in Panel A, the coefficient for lagged NAV return deviation is significantly negative across all Bitcoin fund categories (1×, 2×, -2×, and -1×). For example, the coefficient for the (1×) Bitcoin fund (BITO) is -0.5777, significant at the 1% level. These negative coefficients suggest that part of the lagged NAV return deviation is corrected on the following trading day. The coefficient on the previous day's interest is negative for bull funds and positive for bear funds. These signs are consistent with interest charges for leveraged (double) funds and interest income for inverse funds, although none of these coefficients is statistically significant. The coefficients on the fund's target return are positive and statistically significant for the (1×) and (-1×) Bitcoin funds, suggesting that their NAVs are overexposed to spot Bitcoin movements.

Panel B of Table 3 presents the NAV return deviation analysis for Gold funds. Similar to the Bitcoin funds, the lagged NAV return deviation is a significant predictor of current NAV return deviation, with coefficients of -0.4497, -0.6189, and -0.6106 for the (1×), (2×), and (-2×) Gold funds, respectively (all $p < 0.01$). These coefficients indicate that a significant portion of the previous day's NAV deviation is corrected on the following trading day for Gold funds as well. Additionally, the negative coefficients on the fund target return suggest that these funds are underexposed to the spot Gold market.

In summary, Table 3 shows that the lagged NAV return deviations have negative coefficients in predicting current deviations for both Bitcoin and Gold funds, indicating a short-term correction mechanism in NAV tracking.

4.2.2. Inefficiency Return Deviation Analysis

Table 4 presents the analytical results on the inefficiency return deviation, which is defined as the difference between the fund's total return and its NAV return. This analysis aims to identify factors that cause discrepancies between the fund's market price-based total return and its NAV-based total return. All variables are winsorized at the 1% level to mitigate the impact of outliers.

Panel A of Table 4 displays the results for Bitcoin funds. The lagged fund premium, defined as the difference between the fund's market price and its NAV, is a highly significant predictor of inefficiency return deviation across all Bitcoin funds. The coefficients are negative and statistically significant at the 1% level. For example, the coefficient for the (1×) Bitcoin fund (BITO) is -0.8934, indicating that approximately 89% of the lagged fund premium is corrected on the subsequent trading day. The coefficients are -1.0, -0.8, and -0.97 for the (2×), (-2×), and (-1×) Bitcoin funds, respectively.

For the (2×) and (-2×) Bitcoin funds, the coefficient on fund target return is significantly negative, suggesting that higher target returns are associated with negative inefficiency return deviations. This may reflect selling pressure when leveraged fund prices increase sharply and buying pressure when they decrease significantly.

The bid-ask spread has a statistically significant positive coefficient for the (1×) and (2×) Bitcoin funds, implying that market prices tend to rise when the bid-ask spread widens for these bull funds. Other variables—such as fund flow, changes in the Fed Funds Rate, S&P 500 index

returns, VIX index levels, annual inflation, and year-over-year GDP growth—are included as controls. Their significance varies across fund categories.

Panel B of Table 4 shows results for Gold funds. Similarly, the lagged fund premium is a significant predictor of inefficiency return deviation, with coefficients ranging from about -0.99 to -1.13, all statistically significant. The coefficient on fund target return is also significant for Gold funds.

In summary, Table 4 demonstrates that the lagged fund premium is the primary driver of inefficiency return deviations for both Bitcoin and Gold funds, indicating that price discrepancies tend to correct on the following trading day. Fund-specific characteristics, such as turnover and bid-ask spreads, and broader market factors also influence these deviations, though their effects vary by fund.

4.2.3. Daily Total Return Deviation Analysis

Table 5 presents the results of the daily total return deviation analysis, which examines factors influencing the difference between the fund's actual total return and its target return. All variables in these regressions are winsorized at the 1% level to reduce the impact of outliers.

Panel A of Table 5 shows results for Bitcoin funds. The lagged total return deviation is a significant predictor of the current total return deviation across all Bitcoin funds. The coefficients are negative and statistically significant at the 1% level, ranging from -0.57 to -0.68, indicating that most past deviations tend to be corrected in the subsequent trading day. Several market and macroeconomic variables are included in the analysis. For example, the S&P 500 (SPX) index return has a statistically significant positive coefficient for bull Bitcoin funds and a negative coefficient for bear funds. This pattern may reflect investor sentiment to buy bull Bitcoin funds

and sell bear Bitcoin funds when the stock market performs well. Other variables, including changes in the Fed Funds Rate, VIX index, annual inflation, year-over-year nominal GDP growth, fund turnover, bid-ask spread, and fund flow, are also controlled for, though their statistical significance varies across funds.

Panel B of Table 5 displays the results for Gold funds. Similar to Bitcoin funds, the lagged total return deviation significantly predicts the current total return deviation, with coefficients around -0.4 to -0.5 and statistically significant. The coefficient on SPX index return is significantly positive for bull Gold funds and negative for bear Gold funds, which can also be explained by investor sentiment responding to stock market performance.

In summary, Table 5 highlights that past total return deviations are a primary driver of current deviations for both Bitcoin and Gold funds, indicating a corrective mechanism in total return tracking. Market factors also play important roles in explaining these deviations.

4.3. Summary Statistics for Multi-Day Returns

When the holding period extends beyond a single trading day, the target return of an LETF—defined as the daily leveraged target return compounded over the holding period—deviates from its naïve expected return, calculated as the product of the daily leveraged multiple and the cumulative return of the underlying index. This introduces an additional source of deviation, referred to as the compounding deviation, alongside the NAV return deviation and inefficiency deviation identified in the daily case.

Table 6 reports summary statistics for multi-day returns and deviations for Bitcoin and Gold LETFs over 5-day and 20-day horizons, extending the daily results shown in Table 1 to provide insights into how deviations accumulate over longer holding periods. The table

summarizes each fund's actual total return, NAV return, target return, naïve expected return, and corresponding measures of compounding deviation (Fund Target Return minus Fund Naïve expected return), NAV return deviation (Fund NAV Return minus Fund Target Return), inefficiency deviation (Fund Actual total return minus NAV return), and total return deviation (Fund Actual total return minus Fund Naïve expected return).

Panel A details the multi-day return statistics for Bitcoin funds. The mean 5-day total return for the (1×) Bitcoin fund is 1.39%, with a standard deviation of 7.55%. Its mean NAV return deviation is -0.09%, nearly five times the daily NAV return deviation of -0.02% shown in Table 1, Panel A. Its inefficiency deviation averages 0.01%, while the total return deviation is -0.08%. For the (2×) Bitcoin fund, the mean 5-day actual total return is 2.89%, with a notably higher standard deviation of 14.62%, reflecting the amplified volatility of leveraged funds. The average NAV return deviation is -0.63%, close to five times the daily NAV return deviation of -0.11% reported in Table 1, Panel A. For the (-2×) Bitcoin fund, the mean 5-day actual total return is -2.42%, with a standard deviation of 14.02%. The average target return is -2.5%, and the average naïve expected return is -2.34%. This results in an average compounding deviation of -0.16%, with a standard deviation of 2.02%. The median compounding effect is -0.36%.

Panel B presents the corresponding statistics for Gold funds over the same 5-day holding period. The mean fund actual total return for the (1×) Gold fund is 0.30%, with a standard deviation of 2.02%, both substantially lower than those of the (1×) Bitcoin fund shown in Panel A. As with Bitcoin funds, statistics for fund NAV return, target return, and various return deviations are included. The mean fund NAV return deviation for the (1×) Gold fund is -0.01%, much smaller in magnitude than that of the (1×) Bitcoin fund (-0.01% vs. -0.09%). The average compounding

deviation for the (-2×) Gold fund is 0.00%, with a standard deviation of 0.16% and a median of -0.02%. These values are considerably smaller than their (-2×) Bitcoin fund counterparts.

Panels C and D extend the holding period to 20 trading days. Generally, the NAV return deviation accumulates roughly linearly with the number of holding days, while inefficiency does not accumulate. For instance, the average 20-day NAV return deviation for the (1×) Bitcoin fund is -0.42%, approximately 20 times the daily NAV return deviation of -0.02% shown in Table 1. The average 20-day compounding deviation for the (-2×) Bitcoin fund is -0.63%, with a standard deviation of 8.78% and a median of -2.66%. These figures are larger in magnitude than their 5-day counterparts in Panel A.

In summary, Table 6 offers a comparative overview of multi-day return characteristics for Bitcoin and Gold LETFs, highlighting notable differences in return magnitude and volatility.

4.4. Analysis of Multi-Day Compounding Deviation

Table 7 presents the results of the multi-day compounding deviation analysis, which examines the factors driving the difference between a fund's target return and its naïve expected return over 5-day or 20-day holding periods. Based on the LETF compounding deviation model of Tang and Xu (2013),³ we hypothesize that the compounding deviation of Bitcoin and Gold LETFs is positively associated with the squared cumulative underlying index return, negatively associated with the variance of daily underlying index returns, and the strength of the compounding effect is positively related to the multiple function (m^2-m).

Given the relatively short sample period, we first present the regression results based on 5-trading-day rolling periods in Panel A and Panel B for the Bitcoin LETFs and Gold LETFs,

³ See Equation (5) in Tang and Xu (2013) for the LETF compounding deviation model.

respectively, followed by 20-trading-day rolling periods in Panel C and Panel D. Panel E and Panel F report the results for the non-overlapping 5-day periods while Panel G and Panel H present results for the non-overlapping 20-day periods.

Panel A shows the results for Bitcoin funds during 5-day holding periods that confirm these hypotheses. For example, in column (1), the coefficient on the squared cumulative index return is 0.0074, and the coefficient on the variance of daily index returns is -0.0398 for the (2×) Bitcoin fund. Both coefficients are statistically significant at the 1% level. The regression's R-squared is 0.994, indicating that these variables almost perfectly explain the variation in compounding deviation. The coefficients for the (−2×) Bitcoin fund are approximately three times those of the (2×) Bitcoin fund. Moreover, the coefficients for the (−1×) Bitcoin fund are similar to those of the (2×) fund. These results provide strong evidence that the strength of the compounding effect is positively related to the multiple function (m^2-m), with the (−2×) fund exhibiting roughly triple the compounding effect relative to the (2×) and (−1×) funds.

Panel B displays results for the (2×) and (−2×) Gold funds over 5-day holding periods. In column (1), the coefficient on the squared cumulative index return is 0.0079, closely matching its Bitcoin counterpart. The coefficient on the variance of daily index returns is -0.0403, also similar to the Bitcoin fund result reported in Panel A. These findings indicate that the coefficients are consistent across the (2×) Bitcoin and (2×) Gold funds, despite differences in the magnitudes of their compounding deviations. This similarity also holds for the (−2×) Bitcoin and (−2×) Gold funds.

As we transition from the 5-day rolling periods in Panel A and Panel B to 20-day rolling periods in Panel C and Panel D, we observe similar coefficients for the squared cumulative index return, while the coefficients for the variance of daily index returns increase roughly fourfold. This pattern is consistent with the LETF compounding deviation model of Tang and Xu (2013), which

predicts that the multiple function (m^2-m) is positively related to the magnitude of the positive coefficient on the squared cumulative index return and the magnitude of the negative coefficient on the variance of daily index returns, with the negative impact of index return variance becoming stronger as the holding period (t) lengthens.

While Panels A–D present the regression results based on rolling 5-day and 20-day windows, Panels E–H report the corresponding estimates using non-overlapping 5-day and 20-day periods. Overall, the signs and relative magnitudes of the coefficients are consistent across the two approaches, confirming the robustness of the compounding deviation model of Tang and Xu (2013). In both sets of panels, the coefficient on the squared cumulative index return remains positive and statistically significant, while the coefficient on the variance of daily index returns is negative, in line with model predictions. Moreover, the strength of the compounding effect continues to scale with the multiple function (m^2-m), with the $(-2\times)$ funds exhibiting roughly triple the impact relative to the $(2\times)$ and $(-1\times)$ funds. The minor differences in coefficients between Panels A–D and Panels E–H underscore the robustness of our findings across both rolling and non-overlapping holding periods.

In summary, Table 7 demonstrates that the squared cumulative index return and the volatility of daily index returns nearly perfectly explain compounding deviations over multiple holding days for both Bitcoin and Gold funds.

4.5. Analysis of Multi-Day Total Return Deviation

Table 8 presents the results of the multi-day total return deviation analysis. For Gold LETFs, the coefficient on the Holding-period Fed Funds interest rate is -1.3045 for the $(2\times)$ fund, consistent with the cost of financing leveraged exposure through longing Gold futures, while the

coefficient is +2.9914 for the ($-2\times$) fund, reflecting interest income earned from shorting Gold futures. These findings align with the traditional cost-of-carry model of Black (1976), under which futures prices embed financing and storage costs that accumulate with the holding period.

For Bitcoin LETFs, the estimated coefficients on the Holding-period Fed Funds rate deviate from the systematic patterns implied by the cost-of-carry framework. In Panel A, the coefficient is -7.0738 for the ($2\times$) fund and -15.9031 for the ($-2\times$) fund, which contradicts theoretical predictions of -1 and $+3$ from the cost-of-carry framework. Although financing costs may still influence NAV tracking performance through collateral funding and arbitrage between spot and futures markets, prior research indicates that the Bitcoin futures basis is driven less by interest rates and more by investor sentiment, market volatility, and unregulated market activity (Alexander and Heck, 2020; Lee, El Meslmani, and Switzer, 2020; Makarov and Schoar, 2020; Conlon, Corbet, and Oxley, 2024). Thus, whereas holding-period interest rates provide a systematic explanation for the cumulative total return deviations of Gold LETFs, they do not consistently explain the cumulative total return deviations of Bitcoin LETFs.

The coefficients on the squared cumulative index return are uniformly positive and significant across both Bitcoin and Gold LETFs, confirming that total return deviations increase with sustained directional movements in the underlying index. By contrast, the coefficients on the variance of daily index returns are negative, consistent with volatility drag eroding performance relative to leverage objectives. For Bitcoin LETFs, these variance effects are consistently significant with relatively high explanatory power, underscoring the critical role of volatility drag in cryptocurrency-based LETFs. For Gold LETFs, the variance coefficients are generally negative but often less significant, with much lower explanatory power, suggesting that volatility drag,

while present, is a less dominant driver of deviations given the lower volatility of the Gold futures market.

A comparison of the rolling-window results in Panels A–D with the non-overlapping results in Panels E–H confirms the robustness of these patterns. Although the magnitudes are smaller in the non-overlapping case—reflecting reduced serial correlation and smaller sample size—the signs and relative magnitudes remain stable.

5. Conclusion

This paper provides the first systematic analysis of the tracking performance of Bitcoin leveraged ETFs, comparing them with Gold LETFs as a mature reference class. Using the return deviation framework of Tang and Xu (2013, 2016), we decompose total return deviations into compounding, NAV, and inefficiency components and assess their behavior across daily and multi-day horizons.

On a daily basis, Bitcoin LETFs track their leveraged multiples closely, with insignificant beta deviations, whereas Gold LETFs display significant beta deviations, indicating underexposure to the underlying gold index. Further decomposition of daily total return deviations shows that NAV deviations are the main source of daily tracking error, while inefficiency deviations are small and short-lived. For Bitcoin LETFs, NAV deviations are negative for leveraged funds and positive for inverse funds, consistent with trading frictions and nonsynchronous pricing, and their standard deviations are relatively large, indicating greater variability. Inefficiency deviations remain economically minor in both markets, though somewhat larger and more volatile for Bitcoin funds, reflecting thinner liquidity and speculative pressures.

Over multi-day horizons, compounding deviation clearly emerges as the dominant driver of return deviations. Both Bitcoin and Gold ETFs exhibit divergence from their naïve expected returns as holding periods extend from five to twenty days, but the compounding effect is far more pronounced for Bitcoin products, reflecting the extreme volatility of the underlying asset. The empirical results are consistent with the predictions of the compounding deviation framework of Tang and Xu (2013): volatility drag grows with leverage and with the length of the holding period, such that multi-day actual returns diverge systematically from their leverage targets. By contrast, NAV deviations accumulate more gradually and remain secondary in importance, small for Gold ETFs and broadly aligned with the cost-of-carry model, but erratic and less predictable for Bitcoin ETFs, reflecting sentiment, liquidity frictions, and unregulated markets. Inefficiency deviations are transitory and economically minor in both markets, though they are somewhat larger and more volatile for Bitcoin ETFs.

Overall, these results provide new insights into the tracking performance of cryptocurrency leveraged ETFs. For investors and portfolio managers, the findings confirm that Bitcoin ETFs can deliver their stated leveraged multiple on a daily basis but are poorly suited for multi-day exposure due to compounding drag and unpredictable NAV deviations. They may be useful for short-term tactical exposure but are unsuitable for buy-and-hold strategies. For regulators, the analysis highlights the vulnerabilities of cryptocurrency-based ETFs and the importance of enhanced disclosure and oversight. As digital asset markets evolve and mature, continued research into these structural dynamics will be essential for guiding investment strategies and informing sound regulatory policy.

References

- Alexander, C., and Heck, D. F. 2020. Price discovery in Bitcoin: The impact of unregulated markets. *Journal of Financial Stability* 50: 100776.
- Black, F. 1976. The pricing of commodity contracts. *Journal of Financial Economics* 3(1–2): 167–179.
- Conlon, T., Corbet, S., and Oxley, L. 2024. Investor sentiment, unexpected inflation, and Bitcoin basis. *Journal of Futures Markets* 44(11): 1807–1831.
- Gkillas, K., and F. Longin. 2019. Is Bitcoin the new digital gold? Evidence from extreme price movements in financial markets. SSRN Working Paper No. 3245571.
- Guo, K., and Leung, T. 2016. Understanding the tracking errors of commodity leveraged ETFs. *Quantitative Finance* 16(9): 1431–1447.
- Lee, S., El Meslmani, N., and Switzer, L. N. 2020. Pricing efficiency and arbitrage in the Bitcoin spot and futures markets. *Research in International Business and Finance* 53: 101200.
- Makarov, I., and Schoar, A. 2020. Trading and arbitrage in cryptocurrency markets. *Journal of Financial Economics* 135(2): 293–319.
- Petajisto, A. 2017. Inefficiencies in the pricing of exchange-traded funds. *Review of Financial Studies* 30(4): 982–1018.
- Tang, H., and Xu, X. E. 2013. Solving the return deviation conundrum of leveraged exchange-traded funds. *Journal of Financial and Quantitative Analysis* 48(1): 309–342.
- Tang, H., and Xu, X. E. 2016. Tracking performance of leveraged energy exchange-traded funds. *Journal of Derivatives* 23(3): 37–60.
- Tang, H., Xie, K., and Xu, X. E. 2025. Cryptocurrency ETFs vs. nonredeemable investment trusts: An in-depth analysis. *International Review of Economics and Finance* 102: 104264.

Table 1: Summary Statistics for Daily Variables

This table presents the summary statistics of daily returns, tracking performance, its components, and other daily variables. There are 636 observations for (1× Bitcoin), (-1× Bitcoin), and all Gold funds. There are 381 observations for (2× Bitcoin) and 189 observations for (-2× Bitcoin) funds. None of the variables are winsorized.

Panel A: Bitcoin Funds

		<u>2 × Bitcoin Fund</u>		<u>(-2) × Bitcoin</u>		<u>(-1) × Bitcoin</u>			
Variable	Row Label	Mean	<i>S. D.</i>	Mean	<i>S. D.</i>	Mean	<i>S. D.</i>	Mean	<i>S. D.</i>
Fund Actual Total Return	(1)	0.28	3.56	0.58	6.68	-0.44	7.06	-0.26	3.55
Fund NAV Return	(2)	0.28	3.56	0.58	6.73	-0.44	7.06	-0.26	3.55
Fund Target Return	(3)	0.29	3.35	0.69	6.34	-0.49	6.77	-0.29	3.35
Fund Underlying Index Return	(4)	0.29	3.35	0.34	3.17	0.24	3.39	0.29	3.35
Fund Total Return Deviation	(1) - (3)	-0.01	0.97	-0.11	1.83	0.05	1.89	0.03	0.96
Fund NAV Return Deviation	(2) - (3)	-0.02	0.96	-0.11	1.84	0.05	1.86	0.03	0.95
Fund Inefficiency Return Deviation	(1) - (2)	0.00	0.16	-0.01	0.37	0.00	0.23	0.00	0.17
Fund Price		18.09	5.50	30.38	14.81	33.06	11.99	97.92	59.41
Fund NAV		18.09	5.50	30.36	14.81	33.08	12.00	97.92	59.41
Fund Assets Under Management		1389	708	1073	1035	24.95	14.56	84.78	22.51
Fund Flow		1.40	23.36	5.67	21.29	0.47	2.79	0.42	3.90
Lagged Fund Valuation Premium		0.00	0.12	0.08	0.28	-0.03	0.17	-0.01	0.12
Fund Turnover		0.16	0.12	0.18	0.15	1.09	0.57	0.54	0.53
Fund Bid-Ask Spread		0.07	0.03	0.14	0.11	0.18	0.08	0.09	0.04

Panel B: Gold Funds

				<u>2 × Gold Fund</u>		<u>(-2) × Gold Fund</u>	
Variable		Mean	<i>S. D.</i>	Mean	<i>S. D.</i>	Mean	<i>S. D.</i>
Fund Actual Total Return	(1)	0.06	<i>0.91</i>	0.09	<i>1.83</i>	-0.07	<i>1.82</i>
Fund NAV Return	(2)	0.06	<i>0.86</i>	0.09	<i>1.80</i>	-0.07	<i>1.80</i>
Fund Target Return	(3)	0.06	<i>0.91</i>	0.12	<i>1.83</i>	-0.12	<i>1.83</i>
Fund Underlying Index Return	(4)	0.06	<i>0.91</i>	0.06	<i>0.91</i>	0.06	<i>0.91</i>
Fund Total Return Deviation	(1) - (3)	0.00	<i>0.13</i>	-0.03	<i>0.27</i>	0.05	<i>0.26</i>
Fund NAV Return Deviation	(2) - (3)	0.00	<i>0.73</i>	-0.03	<i>0.76</i>	0.05	<i>0.76</i>
Fund Inefficiency Return Deviation	(1) - (2)	0.00	<i>0.73</i>	0.00	<i>0.75</i>	0.00	<i>0.75</i>
Fund Price		192.54	<i>27.72</i>	66.61	<i>15.20</i>	26.75	<i>5.83</i>
Fund NAV		192.50	<i>27.69</i>	66.61	<i>15.19</i>	26.75	<i>5.83</i>
Fund Assets Under Management		59412	<i>7170</i>	209	<i>47</i>	17.85	<i>6.22</i>
Fund Flow		-15.81	<i>175.61</i>	-0.09	<i>2.46</i>	-0.02	<i>0.76</i>
Lagged Fund Valuation Premium		0.02	<i>0.51</i>	0.01	<i>0.52</i>	-0.01	<i>0.52</i>
Fund Turnover		0.02	<i>0.01</i>	0.05	<i>0.03</i>	0.16	<i>0.10</i>
Fund Bid-Ask Spread		0.01	<i>0.00</i>	0.06	<i>0.03</i>	0.06	<i>0.02</i>

Note: See Appendix Table A1 for variable definitions.

See Appendix Table A2 for the specifications of each Bitcoin and Gold fund.

Table 2: Daily Beta Estimation

This table presents the estimation results of the single-index model for the exposure of Bitcoin and Gold funds to their respective underlying index using daily observations and heteroskedasticity-robust standard errors.

Panel A: Market Price Beta of Bitcoin Funds

Dep. Var.	Daily Actual Total Return based on Fund Market Price			
Fund Multiple	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	SBIT	BITI
	(1)	(2)	(3)	(4)
Spot Bitcoin Return	1.0231*** (0.016)	2.0255*** (0.036)	-2.0080*** (0.047)	-1.0222*** (0.015)
Constant	-0.0209 (0.039)	-0.1189 (0.095)	0.0525 (0.138)	0.0380 (0.039)
Observations	636	381	189	636
R-squared	0.926	0.925	0.928	0.927
F-statistic	4333	3211	1800	4530
Adjusted R-squared	0.926	0.925	0.928	0.927

Panel B: Market Price Beta of Gold Funds

Dep. Var.	Daily Actual Total Return based on Fund Market Price		
Fund Multiple	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL
	(1)	(2)	(3)
Spot Gold Return	0.9853*** (0.008)	1.9798*** (0.016)	-1.9727*** (0.016)
Constant	-0.0006 (0.005)	-0.0285*** (0.010)	0.0482*** (0.010)
Observations	636	636	636
R-squared	0.981	0.979	0.979
F-statistic	15150	15581	14875
Adjusted R-squared	0.981	0.979	0.979

Panel C: Market Price Beta Deviation of Bitcoin Funds

Dep. Var.	Daily Actual Total Return - Target Return			
Fund Multiple	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	BITU	BITI
	(1)	(2)	(3)	(4)
Spot Bitcoin Return	0.0231 (0.016)	0.0255 (0.036)	0.0084 (0.047)	-0.0222 (0.015)
Constant	-0.0209 (0.039)	-0.1189 (0.095)	-0.0746 (0.136)	0.0380 (0.039)
Observations	636	381	189	636
R-squared	0.006	0.002	0.000	0.006
F-statistic	2.217	0.510	0.0315	2.133
Adjusted R-squared	0.004	-0.001	-0.005	0.004

Panel D: Market Price Beta Deviation of Gold Funds

Dep. Var.	Daily Actual Total Return - Target Return		
Fund Multiple	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL
	(1)	(2)	(3)
Spot Gold Return	-0.0147* (0.008)	-0.0202 (0.016)	0.0273* (0.016)
Constant	-0.0006 (0.005)	-0.0285*** (0.010)	0.0482*** (0.010)
Observations	636	636	636
R-squared	0.011	0.005	0.009
F-statistic	3.369	1.615	2.849
Adjusted R-squared	0.009	0.003	0.007

Panel E: NAV Beta of Bitcoin Funds

Dep. Var.	Daily NAV Return			
Fund Multiple	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	BITU	BITI
	(1)	(2)	(3)	(4)
Spot Bitcoin Return	1.0239*** (0.015)	2.0421*** (0.036)	2.0232*** (0.046)	-1.0227*** (0.015)
Constant	-0.0222 (0.038)	-0.1196 (0.095)	-0.0718 (0.136)	0.0379 (0.038)
Observations	636	381	189	636
R-squared	0.928	0.926	0.931	0.929
F-statistic	4648	3160	1921	4642
Adjusted R-squared	0.928	0.926	0.931	0.929

Panel F: NAV Beta of Gold Funds

Dep. Var.	Daily NAV Return		
Fund Multiple	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL
	(1)	(2)	(3)
Spot Gold Return	0.6210*** (0.033)	1.7981*** (0.042)	-1.7984*** (0.042)
Constant	0.0195 (0.025)	-0.0191 (0.029)	0.0382 (0.029)
Observations	636	636	636
R-squared	0.438	0.832	0.832
F-statistic	345.5	1807	1801
Adjusted R-squared	0.437	0.832	0.832

Panel G: NAV Beta Deviation of Bitcoin Funds

Dep. Var.	Daily NAV Return - Target Return			
Fund Multiple	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	BITU	BITI
	(1)	(2)	(3)	(4)
Spot Bitcoin Return	0.0239 (0.015)	0.0421 (0.036)	0.0232 (0.046)	-0.0227 (0.015)
Constant	-0.0222 (0.038)	-0.1196 (0.095)	-0.0718 (0.136)	0.0379 (0.038)
Observations	636	381	189	636
R-squared	0.007	0.005	0.002	0.006
F-statistic	2.531	1.345	0.253	2.287
Adjusted R-squared	0.005	0.002	-0.003	0.004

Panel H: NAV Beta Deviation of Gold Funds

Dep. Var.	Daily NAV Return - Target Return		
Fund Multiple	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL
	(1)	(2)	(3)
Spot Gold Return	-0.3790*** (0.033)	-0.2019*** (0.042)	0.2016*** (0.042)
Constant	0.0195 (0.025)	-0.0191 (0.029)	0.0382 (0.029)
Observations	636	636	636
R-squared	0.225	0.059	0.059
F-statistic	128.7	22.79	22.63
Adjusted R-squared	0.224	0.058	0.058

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Note: See Appendix Table A1 for variable definitions.

See Appendix Table A2 for the specifications of each Bitcoin and Gold fund.

Table 3: Daily NAV Return Deviation Analysis

This table presents the results from the daily NAV return deviation analysis using OLS regressions with heteroskedasticity-robust standard errors. All variables are winsorized at 1% level on both sides.

Panel A: Bitcoin Funds

	Dependent Variable = NAV Return Deviation			
Fund Multiple	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	SBIT	BITI
	(1)	(2)	(3)	(4)
Interest from Previous	-2.5946	-3.4934	8.8717	2.7482
Trading Day	(2.202)	(5.177)	(7.507)	(2.186)
Lagged NAV Return	-0.5777***	-0.6309***	-0.7147***	-0.5850***
Deviation	(0.041)	(0.051)	(0.059)	(0.040)
Fund Target Return	0.0318***	0.0229	0.0070	0.0321***
	(0.012)	(0.014)	(0.016)	(0.012)
Fund Flow	-0.0004	-0.0063	-0.0181*	-0.0135
	(0.002)	(0.006)	(0.010)	(0.010)
VIX Index Level	0.0057	0.0126	0.0204	-0.0070
	(0.006)	(0.026)	(0.027)	(0.006)
Constant	-0.0860	-0.2791	-0.5319	0.1438
	(0.128)	(0.426)	(0.474)	(0.128)
Observations	636	380	188	635
R-squared	0.340	0.395	0.461	0.346
F-statistic	44.92	32.59	30.33	47.82
Adjusted R-squared	0.335	0.387	0.446	0.341

Panel B: Gold Funds

	Dependent Variable = NAV Return Deviation		
Fund Multiple	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL
	(1)	(2)	(3)
Interest from Previous	-2.4190	0.5423	-0.2952
Trading Day	(1.784)	(1.574)	(1.558)
Lagged NAV Return	-0.4669***	-0.5543***	-0.5537***
Deviation	(0.034)	(0.044)	(0.045)
Fund Target Return	-0.3650***	-0.1042***	-0.1052***
	(0.026)	(0.016)	(0.016)
Fund Flow	-0.0001	0.0168	0.0500
	(0.000)	(0.013)	(0.033)
VIX Index Level	-0.0036	0.0013	-0.0025
	(0.005)	(0.005)	(0.005)
Constant	0.1251	-0.0672	0.1170
	(0.107)	(0.105)	(0.106)
Observations	636	636	636
R-squared	0.445	0.360	0.361
F-statistic	88.87	44.51	44.25
Adjusted R-squared	0.441	0.355	0.356

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Note: See Appendix Table A1 for variable definitions.

See Appendix Table A2 for the specifications of each Bitcoin and Gold fund.

Table 4: Daily Inefficiency Return Deviation Analysis

This table presents the results from the daily inefficiency return deviation analysis using OLS with heteroskedasticity robust standard errors. All variables are winsorized at 1% level on both sides.

Panel A: Bitcoin Funds

	Dependent Variable = Inefficiency Return Deviation			
Fund Multiple	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	SBIT	BITI
	(1)	(2)	(3)	(4)
Lagged Fund Premium	-0.8934*** (0.046)	-1.0089*** (0.069)	-0.8001*** (0.039)	-0.9720*** (0.044)
Fund Target Return	0.0017 (0.002)	-0.0092*** (0.002)	-0.0097*** (0.002)	0.0008 (0.002)
Fund Turnover	0.0516 (0.048)	-0.1630 (0.121)	0.0766 (0.063)	-0.0112 (0.012)
Fund Bid Ask Spread	0.5803*** (0.176)	0.7865*** (0.098)	0.1212 (0.184)	0.0477 (0.123)
Fund Fund Flow	-0.0000 (0.000)	0.0007 (0.001)	0.0002 (0.001)	-0.0013 (0.001)
Change in Fed Funds Rate	0.0466 (0.156)	11.8512** (5.052)	-0.1815 (0.274)	-0.0935 (0.108)
Constant	-0.0425*** (0.015)	-0.0063 (0.026)	0.0235 (0.027)	-0.0064 (0.010)
Observations	636	381	189	636
R-squared	0.440	0.520	0.490	0.479
F-statistic	64.75	54.09	75.64	86.99
Adjusted R-squared	0.435	0.512	0.473	0.474

Panel B: Gold Funds

	Dependent Variable = Inefficiency Return Deviation			
Fund Multiple	×1	×2	×(-2)	×1
Fund Ticker	GLD	UGL	GLL	GLD
	(1)	(2)	(3)	(4)
Lagged Fund Premium	-0.9859*** (0.034)	-1.1311*** (0.035)	-1.1239*** (0.034)	-0.9890*** (0.034)
Fund Target Return	0.3143*** (0.022)	0.0835*** (0.014)	0.0738*** (0.013)	0.2945*** (0.021)
Fund Turnover	3.1039 (2.105)	1.3254 (0.880)	-0.6210*** (0.202)	4.0973** (2.079)
Fund Bid-Ask Spread	-1.5431 (3.872)	-0.5750 (0.807)	0.2164 (0.801)	-3.1810 (4.732)
Fund Fund Flow	0.0003*** (0.000)	-0.0040 (0.013)	-0.0640** (0.029)	0.0003*** (0.000)
Change in Fed Funds Rate	0.5212 (0.358)	0.2305 (0.321)	-0.0673 (0.382)	0.5446 (0.430)
Constant	-0.0499 (0.052)	-0.0351 (0.054)	0.0838 (0.054)	0.0431 (0.144)
Observations	636	636	636	636
R-squared	0.684	0.574	0.587	0.696
F-statistic	211.6	195.2	188.5	136.0
Adjusted R-squared	0.681	0.570	0.583	0.691

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Note: See Appendix Table A1 for variable definitions.

See Appendix Table A2 for the specifications of each Bitcoin and Gold fund.

Table 5: Daily Total Return Deviation Analysis

This table presents the results from the daily total return deviation analysis using OLS with heteroskedasticity robust standard errors. All variables are winsorized at 1% level on both sides.

Panel A: Bitcoin Funds

	Dependent Variable = Total Return Deviation							
Fund Multiple	×1	×2	×(-2)	×(-1)	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	SBIT	BITI	BITO	BITX	SBIT	BITI
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Lagged Total Return Deviation	-0.5733*** (0.033)	-0.6175*** (0.041)	-0.6808*** (0.058)	-0.5814*** (0.033)	-0.5788*** (0.042)	-0.6269*** (0.051)	-0.6867*** (0.062)	-0.5967*** (0.038)
Fund Target Return	0.0305*** (0.010)	0.0128 (0.012)	-0.0041 (0.016)	0.0313*** (0.010)	0.0218 (0.013)	0.0061 (0.014)	-0.0257 (0.020)	0.0205 (0.012)
Change in Fed Funds Rate					1.5005* (0.870)	52.1609 (49.382)	1.2157 (1.818)	-1.1791 (0.746)
SPX Index return					0.1140*** (0.036)	0.2429** (0.117)	-0.4273*** (0.143)	-0.1140*** (0.035)
VIX Index					0.0093 (0.009)	0.0335 (0.028)	-0.0427 (0.041)	-0.0182* (0.010)
Annual Inflation					-0.0239 (0.060)	0.0768 (0.213)	-0.1601 (0.319)	0.0360 (0.057)
GDP Year over Year Nominal					0.0241 (0.068)	0.0277 (0.190)	0.1239 (0.385)	-0.0199 (0.066)
Fund Turnover					-0.5534 (0.449)	-0.5667 (0.739)	-0.0620 (0.259)	0.2354** (0.120)
Fund Bid Ask Spread					0.5747 (1.699)	-0.6947 (0.901)	-0.7280 (1.736)	-1.0577 (1.027)
Fund Flow					-0.0001 (0.002)	-0.0066 (0.005)	-0.0697 (0.064)	-0.0181* (0.010)
Constant	-0.0289 (0.030)	-0.1896*** (0.073)	0.0518 (0.105)	0.0626** (0.030)	-0.2225 (0.269)	-0.8861 (1.267)	0.8023 (2.243)	0.3522 (0.288)
Observations	636	380	188	635	636	380	188	635
R-squared	0.331	0.375	0.430	0.341	0.351	0.395	0.461	0.369
F-statistic	156.9	113.0	69.70	163.5	24.04	17.30	13.41	28.37
Adjusted R-squared	0.330	0.373	0.427	0.340	0.341	0.379	0.431	0.359

Panel B: Gold Funds

	Dependent Variable = Total Return Deviation					
Fund Multiple	×1	×2	×(-2)	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL	GLD	UGL	GLL
	(1)	(2)	(3)	(4)	(5)	(6)
Lagged Total Return Deviation	-0.4768*** (0.035)	-0.4320*** (0.036)	-0.4275*** (0.036)	-0.4763*** (0.043)	-0.4270*** (0.046)	-0.4309*** (0.045)
Fund Target Return	-0.0064 (0.005)	-0.0032 (0.005)	-0.0048 (0.005)	-0.0100* (0.006)	-0.0049 (0.006)	-0.0065 (0.006)
Change in Fed Funds Rate				0.0047 (0.147)	-0.1616 (0.276)	0.1489 (0.383)
SPX Index return				0.0131** (0.005)	0.0215* (0.011)	-0.0180* (0.010)
VIX Index				0.0015 (0.002)	0.0039 (0.003)	-0.0031 (0.003)
Annual Inflation				-0.0002 (0.008)	0.0073 (0.017)	-0.0101 (0.017)
GDP Year over Year Nominal				-0.0021 (0.008)	-0.0144 (0.019)	0.0101 (0.018)
Fund Turnover				0.0384 (0.553)	-0.1853 (0.494)	-0.0119 (0.107)
Fund Bid Ask Spread				-0.7181 (1.157)	-0.0409 (0.362)	0.2649 (0.367)
Fund Flow				0.0000 (0.000)	0.0090* (0.005)	0.0033 (0.014)
Constant	-0.0008 (0.004)	-0.0399*** (0.009)	0.0681*** (0.009)	-0.0089 (0.036)	-0.0304 (0.093)	0.0847 (0.086)
Observations	636	636	636	636	636	636
R-squared	0.232	0.188	0.186	0.243	0.201	0.197
F-statistic	95.45	73.29	72.19	14.20	10.55	10.51
Adjusted R-squared	0.231	0.187	0.185	0.231	0.188	0.184

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Note: See Appendix Table A1 for variable definitions.

See Appendix Table A2 for the specifications of each Bitcoin and Gold fund.

Table 6: Summary Statistics for Multiday Returns and Deviations

This table reports the summary statistics of multi-day returns and return deviations. None of the variables are winsorized.

Panel A: Five-trading-day Bitcoin Fund Returns (632 observations for $(1\times)$ and $(-1\times)$ funds; 377 for $(2\times)$, and 185 for $(-2\times)$ Bitcoin funds)

Variable	<u>$2 \times$ Bitcoin Fund</u>			<u>$(-2) \times$ Bitcoin Fund</u>			<u>$(-1) \times$ Bitcoin Fund</u>		
	Mean	<i>S. D.</i>	Median	Mean	<i>S. D.</i>	Median	Mean	<i>S. D.</i>	Median
Fund Actual Total Return	2.89	<i>14.62</i>	1.08	-2.42	<i>14.02</i>	-2.78	-1.37	<i>7.26</i>	-0.85
Fund NAV Return	2.89	<i>14.62</i>	1.21	-2.42	<i>14.06</i>	-2.80	-1.37	<i>7.24</i>	-0.88
Fund Target Return	3.52	<i>14.52</i>	2.38	-2.50	<i>14.03</i>	-2.78	-1.50	<i>7.02</i>	-1.05
Fund Naïve Expected Return	3.51	<i>14.19</i>	2.60	-2.34	<i>14.59</i>	-1.54	-1.47	<i>7.32</i>	-0.74
Fund Compounding Deviation	0.01	<i>0.72</i>	-0.08	-0.16	<i>2.02</i>	-0.36	-0.03	<i>0.79</i>	-0.08
Fund NAV Return Deviation	-0.63	<i>1.86</i>	-0.61	0.08	<i>1.73</i>	0.20	0.13	<i>0.95</i>	0.15
Fund Inefficiency Return Deviation	0.00	<i>0.38</i>	0.01	0.00	<i>0.23</i>	0.02	0.00	<i>0.17</i>	0.00
Fund Total Return Deviation	-0.62	<i>1.96</i>	-0.65	-0.08	<i>2.96</i>	-0.07	0.10	<i>1.30</i>	0.11

Panel B: Five-trading-day Gold Fund Returns (632 observations for all Gold funds)

Variable	<u>$2 \times$ Gold Fund</u>			<u>$(-2) \times$ Gold Fund</u>		
	Mean	<i>S. D.</i>	Median	Mean	<i>S. D.</i>	Median
Fund Actual Total Return	0.46	<i>4.08</i>	0.16	-0.36	<i>4.02</i>	-0.19
Fund NAV Return	0.45	<i>4.04</i>	0.27	-0.36	<i>3.98</i>	-0.29
Fund Target Return	0.61	<i>4.06</i>	0.34	-0.61	<i>3.99</i>	-0.49
Fund Naïve Expected Return	0.61	<i>4.04</i>	0.36	-0.61	<i>4.04</i>	-0.36
Fund Compounding Deviation	0.00	<i>0.05</i>	-0.01	0.00	<i>0.16</i>	-0.02
Fund NAV Return Deviation	-0.16	<i>0.74</i>	-0.15	0.25	<i>0.75</i>	0.24
Fund Inefficiency Return Deviation	0.00	<i>0.74</i>	-0.01	0.00	<i>0.74</i>	0.00
Fund Total Return Deviation	-0.15	<i>0.28</i>	-0.16	0.25	<i>0.32</i>	0.26

Panel C: Twenty-trading-day Bitcoin Fund Returns (617 observations for (1×) and (-1×) funds; 362 for (2×) , and 170 for (-2×) Bitcoin funds)

Variable	<u>2 × Bitcoin Fund</u>			<u>(-2) × Bitcoin Fund</u>			<u>(-1) × Bitcoin Fund</u>		
	Mean	<i>S. D.</i>	Median	Mean	<i>S. D.</i>	Median	Mean	<i>S. D.</i>	Median
Fund Actual Total Return	13.40	<i>34.09</i>	5.55	-11.82	<i>22.54</i>	-13.39	-5.70	<i>12.90</i>	-4.85
Fund NAV Return	13.40	<i>34.08</i>	6.05	-11.82	<i>22.57</i>	-13.34	-5.70	<i>12.89</i>	-4.74
Fund Target Return	16.26	<i>34.90</i>	7.87	-12.12	<i>22.81</i>	-14.16	-6.15	<i>12.78</i>	-5.02
Fund Naïve Expected Return	15.61	<i>31.41</i>	9.62	-11.49	<i>28.96</i>	-9.21	-6.24	<i>15.28</i>	-3.46
Fund Compounding Deviation	0.65	<i>4.37</i>	-0.51	-0.63	<i>8.78</i>	-2.66	0.09	<i>3.44</i>	-0.62
Fund NAV Return Deviation	-2.86	<i>2.90</i>	-2.46	0.29	<i>1.78</i>	0.54	0.45	<i>1.26</i>	0.58
Fund Inefficiency Return Deviation	0.00	<i>0.44</i>	0.01	0.00	<i>0.23</i>	0.02	0.01	<i>0.17</i>	0.01
Fund Total Return Deviation	-2.21	<i>4.48</i>	-2.83	-0.33	<i>9.30</i>	-2.04	0.54	<i>3.64</i>	-0.03

Panel D: Twenty-trading-day Gold Fund Returns (617 observations for all Gold funds)

Variable	<u>2 × Gold Fund</u>			<u>(-2) × Gold Fund</u>		
	Mean	<i>S. D.</i>	Median	Mean	<i>S. D.</i>	Median
Fund Actual Total Return	2.12	<i>7.78</i>	1.25	-1.77	<i>7.43</i>	-1.64
Fund NAV Return	2.12	<i>7.69</i>	1.47	-1.78	<i>7.37</i>	-1.60
Fund Target Return	2.74	<i>7.78</i>	1.98	-2.76	<i>7.38</i>	-2.62
Fund Naïve Expected Return	2.75	<i>7.68</i>	2.12	-2.75	<i>7.68</i>	-2.12
Fund Compounding Deviation	-0.01	<i>0.20</i>	-0.05	-0.01	<i>0.60</i>	-0.15
Fund NAV Return Deviation	-0.62	<i>0.81</i>	-0.60	0.98	<i>0.79</i>	1.00
Fund Inefficiency Return Deviation	0.00	<i>0.78</i>	-0.01	0.01	<i>0.72</i>	0.00
Fund Total Return Deviation	-0.63	<i>0.38</i>	-0.65	0.98	<i>0.68</i>	0.92

Note: See Appendix Table A1 for variable definitions.

See Appendix Table A2 for the specifications of each Bitcoin and Gold fund.

Table 7: Multi-day Compounding Deviation Analysis

This table presents the results from the multi-day compounding return deviation analysis using OLS with heteroskedasticity robust standard errors. None of the variables are winsorized.

Panel A: Bitcoin Leveraged Funds in Rolling 5-trading-day Periods

Dependent Variable = Compounding Return Deviation in 5 Trading Days			
Fund Multiple	×2	×(-2)	×(-1)
Fund Ticker	BITX	SBIT	BITI
	(1)	(2)	(3)
Squared Cumulative	0.0074***	0.0214***	0.0071***
Underlying Index Return	(0.000)	(0.001)	(0.000)
Variance of Daily	-0.0398***	-0.1189***	-0.0385***
Underlying Index Returns	(0.001)	(0.003)	(0.001)
Constant	0.0158**	0.0885***	0.0135***
	(0.007)	(0.026)	(0.005)
Observations	377	185	632
R-squared	0.994	0.985	0.993
F-statistic	3761	667.3	2154

Panel B: Gold Leveraged Funds in Rolling 5-trading-day Periods

Dependent Variable = Compounding Return Deviation in 5 Trading Days		
Fund Multiple	×2	×(-2)
Fund Ticker	UGL	GLL
	(1)	(2)
Squared Cumulative	0.0079***	0.0234***
Underlying Index Return	(0.000)	(0.000)
Variance of Daily	-0.0403***	-0.1190***
Underlying Index Returns	(0.000)	(0.001)
Constant	0.0004	0.0008*
	(0.000)	(0.000)
Observations	632	632
R-squared	0.999	0.999
F-statistic	42003	21047

Panel C: Bitcoin Leveraged Funds in Rolling 20-trading-day Periods

Dependent Variable = Compounding Return Deviation in 20 Trading Days			
Fund Multiple	×2	×(-2)	×(-1)
Fund Ticker	BITX	SBIT	BITI
	(1)	(2)	(3)
Squared Cumulative	0.0083***	0.0222***	0.0071***
Underlying Index Return	(0.000)	(0.001)	(0.000)
Variance of Daily	-0.2041***	-0.5986***	-0.1722***
Underlying Index Returns	(0.011)	(0.038)	(0.008)
Constant	0.2057**	1.0118***	0.0771
	(0.088)	(0.280)	(0.072)
Observations	362	170	617
R-squared	0.994	0.985	0.988
F-statistic	2536	1183	609.6

Panel D: Gold Leveraged Funds in Rolling 20-trading-day Periods

Dependent Variable = Compounding Return Deviation in 20 Trading Days		
Fund Multiple	×2	×(-2)
Fund Ticker	UGL	GLL
	(1)	(2)
Squared Cumulative	0.0091***	0.0269***
Underlying Index Return	(0.000)	(0.000)
Variance of Daily	-0.1953***	-0.5626***
Underlying Index Returns	(0.004)	(0.010)
Constant	0.0055*	0.0106*
	(0.003)	(0.006)
Observations	617	617
R-squared	0.998	0.997
F-statistic	19910	6615

Panel E: Bitcoin Leveraged Funds in Nonoverlapping 5-trading-day Periods

Dependent Variable = Compounding Return Deviation in 5 Trading Days			
Fund Multiple	×2	×(-2)	×(-1)
Fund Ticker	BITX	SBIT	BITI
	(1)	(2)	(3)
Squared Cumulative	0.0076***	0.0231***	0.0072***
Underlying Index Return	(0.000)	(0.000)	(0.000)
Variance of Daily	-0.0385***	-0.1177***	-0.0363***
Underlying Index Returns	(0.001)	(0.002)	(0.001)
Constant	-0.0006	0.0109	-0.0111
	(0.009)	(0.013)	(0.011)
Observations	75	37	126
R-squared	0.993	0.997	0.993
F-statistic	6365	2830	1899

Panel F: Gold Leveraged Funds in Nonoverlapping 5-trading-day Periods

Dependent Variable = Compounding Return Deviation in 5 Trading Days		
Fund Multiple	×2	×(-2)
Fund Ticker	UGL	GLL
	(1)	(2)
Squared Cumulative	0.0079***	0.0234***
Underlying Index Return	(0.000)	(0.000)
Variance of Daily	-0.0402***	-0.1186***
Underlying Index Returns	(0.000)	(0.001)
Constant	0.0003	0.0005
	(0.000)	(0.001)
Observations	126	126
R-squared	0.999	0.999
F-statistic	31346	13102

Panel G: Bitcoin Leveraged Funds in Nonoverlapping 20-trading-day Periods

Dependent Variable = Compounding Return Deviation in 20 Trading Days

Fund Multiple	×2	×(-2)	×(-1)
Fund Ticker	BITX	SBIT	BITI
	(1)	(2)	(3)
Squared Cumulative	0.0085***	0.0250***	0.0075***
Underlying Index Return	(0.000)	(0.001)	(0.000)
Variance of Daily	-0.2122***	-0.6478***	-0.1828***
Underlying Index Returns	(0.016)	(0.060)	(0.009)
Constant	0.2155	1.3877**	0.1296
	(0.152)	(0.464)	(0.093)
Observations	18	8	31
R-squared	0.994	0.994	0.993
F-statistic	1827	4261	529.7

Panel H: Gold Leveraged Funds in Nonoverlapping 20-trading-day Periods

Dependent Variable = Compounding Return Deviation in 5 Trading Days

Fund Multiple	×2	×(-2)
Fund Ticker	UGL	GLL
	(1)	(2)
Squared Cumulative	0.0091***	0.0270***
Underlying Index Return	(0.000)	(0.000)
Variance of Daily	-0.1984***	-0.5577***
Underlying Index Returns	(0.006)	(0.015)
Constant	0.0078**	0.0058
	(0.004)	(0.011)
Observations	31	31
R-squared	0.998	0.998
F-statistic	12652	4310

Newey-West standard errors in parentheses for Panels A-D and robust standard errors in parentheses for Panels E-H.

*** p<0.01, ** p<0.05, * p<0.1

Note: See Appendix Table A1 for variable definitions.

See Appendix Table A2 for the specifications of each Bitcoin and Gold fund.

Table 8: Multi-day Total Return Deviation Analysis

This table presents the results from the multi-day total cumulative return deviation analysis. None of the variables are winsorized.

Panel A: Bitcoin Leveraged Funds in Rolling 5-trading-day Periods

Dependent Variable = Total Cumulative Return Deviation in 5 Trading Days				
Fund Multiple (m)	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	SBIT	BITI
	(1)	(2)	(3)	(4)
Holding-Period Fed	-1.9870	-7.0738	-15.9031**	3.8330*
Funds Interest	(2.023)	(8.588)	(7.843)	(2.038)
Squared Cumulative	-0.0001	0.0054***	0.0252***	0.0076***
Underlying Index Return	(0.001)	(0.001)	(0.002)	(0.001)
Variance of Daily	0.0020	-0.0301**	-0.1608***	-0.0433***
Underlying Index Returns	(0.007)	(0.012)	(0.021)	(0.007)
Constant	0.0797	0.1259	2.0482***	-0.1830
	(0.213)	(0.879)	(0.734)	(0.216)
Observations	632	377	185	632
R-squared	0.003	0.076	0.661	0.442
F-statistic	0.361	8.004	66.46	103.1

Panel B: Gold Leveraged Funds in Rolling 5-trading-day Periods

Dependent Variable = Total Cumulative Return Deviation in 5 Trading Days			
Fund Multiple (m)	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL
	(1)	(2)	(3)
Holding Period Fed Fund	-0.0852	-1.3045**	2.9914***
Interest	(0.205)	(0.543)	(0.518)
Squared Cumulative	-0.0009	0.0064**	0.0251***
Underlying Index Return	(0.001)	(0.003)	(0.002)
Variance of Daily	0.0099	0.0027	-0.1256***
Underlying Index Returns	(0.011)	(0.022)	(0.025)
Constant	-0.0048	-0.0606	-0.0231
	(0.023)	(0.057)	(0.055)
Observations	632	632	632
R-squared	0.005	0.038	0.313
F-statistic	0.480	4.923	47.15

Panel C: Bitcoin Leveraged Funds in Rolling 20-trading-day Periods

Dependent Variable = Total Cumulative Return Deviation in 5 Trading Days				
Fund Multiple (m)	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	SBIT	BITI
	(1)	(2)	(3)	(4)
Holding-Period Fed	-3.2977**	-16.1746	-2.1974	5.7007***
Funds Interest	(1.535)	(11.698)	(7.827)	(2.008)
Squared Cumulative	0.0002	0.0063***	0.0232***	0.0069***
Underlying Index Return	(0.000)	(0.001)	(0.001)	(0.000)
Variance of Daily	-0.0226*	-0.2582***	-0.7009***	-0.1808***
Underlying Index Returns	(0.013)	(0.063)	(0.088)	(0.018)
Constant	1.0017	5.1799	3.1621	-1.4176
	(0.705)	(4.841)	(3.005)	(0.884)
Observations	617	362	170	617
R-squared	0.052	0.613	0.950	0.881
F-statistic	2.228	60.26	745.2	237.9

Panel D: Gold Leveraged Funds in Rolling 20-trading-day Periods

Dependent Variable = Total Cumulative Return Deviation in 5 Trading Days			
Fund Multiple (m)	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL
	(1)	(2)	(3)
Holding Period Fed Fund	-0.0154	-1.4645***	3.0035***
Interest	(0.069)	(0.204)	(0.313)
Squared Cumulative	0.0002	0.0093***	0.0249***
Underlying Index Return	(0.000)	(0.001)	(0.001)
Variance of Daily	0.0056	-0.0712	-0.5660***
Underlying Index Returns	(0.016)	(0.044)	(0.051)
Constant	-0.0344	-0.1781**	-0.0756
	(0.030)	(0.083)	(0.134)
Observations	617	617	617
R-squared	0.002	0.392	0.811
F-statistic	0.484	70.08	357.9

Panel E: Bitcoin Leveraged Funds in Nonoverlapping 5-trading-day Periods

Dependent Variable = Total Cumulative Return Deviation in 5 Trading Days				
Fund Multiple (m)	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	SBIT	BITI
	(1)	(2)	(3)	(4)
Holding-Period Fed	-2.5615	-19.8919	-16.8516	4.1675
Funds Interest	(3.365)	(22.365)	(20.801)	(3.470)
Squared Cumulative	0.0008	0.0102***	0.0235***	0.0071***
Underlying Index Return	(0.001)	(0.002)	(0.005)	(0.001)
Variance of Daily	0.0048	-0.0055	-0.1730***	-0.0472***
Underlying Index Returns	(0.006)	(0.023)	(0.034)	(0.006)
Constant	0.0531	0.9344	2.4382	-0.1342
	(0.324)	(2.379)	(2.327)	(0.333)
Observations	126	75	37	126
R-squared	0.021	0.208	0.405	0.459
F-statistic	0.799	12.92	10.75	36.66

Panel F: Gold Leveraged Funds in Nonoverlapping 5-trading-day Periods

Dependent Variable = Total Cumulative Return Deviation in 5 Trading Days			
Fund Multiple (m)	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL
	(1)	(2)	(3)
Holding Period Federal	-0.2418	-1.3971*	3.1569***
Funds Interest	(0.384)	(0.813)	(0.772)
Squared Cumulative	0.0016	0.0137***	0.0188***
Underlying Index Return	(0.002)	(0.005)	(0.004)
Variance of Daily	-0.0048	-0.0220	-0.0905**
Underlying Index Returns	(0.022)	(0.044)	(0.041)
Constant	0.0099	-0.0649	-0.0383
	(0.040)	(0.083)	(0.080)
Observations	126	126	126
R-squared	0.008	0.104	0.265
F-statistic	0.308	3.841	12.56

Panel G: Bitcoin Leveraged Funds in Nonoverlapping 20-trading-day Periods

Dependent Variable = Total Cumulative Return Deviation in 5 Trading Days				
Fund Multiple (m)	×1	×2	×(-2)	×(-1)
Fund Ticker	BITO	BITX	SBIT	BITI
	(1)	(2)	(3)	(4)
Holding-Period Fed	-3.6225**	-3.1923	-12.4873	6.7709***
Funds Interest	(1.693)	(25.124)	(24.587)	(1.836)
Squared Cumulative	-0.0001	0.0079***	0.0250***	0.0075***
Underlying Index Return	(0.001)	(0.002)	(0.004)	(0.000)
Variance of Daily	-0.0369	-0.2429	-0.7682*	-0.1709***
Underlying Index Returns	(0.029)	(0.180)	(0.331)	(0.026)
Constant	1.3970	-0.7739	7.9606	-2.0903**
	(0.994)	(10.206)	(9.878)	(0.965)
Observations	31	18	8	31
R-squared	0.081	0.633	0.902	0.883
F-statistic	1.530	8.113	35.31	139.4

Panel H: Gold Leveraged Funds in Nonoverlapping 20-trading-day Periods

Dependent Variable = Total Cumulative Return Deviation in 5 Trading Days			
Fund Multiple (m)	×1	×2	×(-2)
Fund Ticker	GLD	UGL	GLL
	(1)	(2)	(3)
Holding Period Federal	-0.1563	-1.6099***	3.0432***
Funds Interest	(0.224)	(0.493)	(0.562)
Squared Cumulative	0.0000	0.0093**	0.0252***
Underlying Index Return	(0.002)	(0.004)	(0.002)
Variance of Daily	-0.0423	-0.1162	-0.5460***
Underlying Index Returns	(0.068)	(0.129)	(0.126)
Constant	0.0582	-0.0880	-0.1126
	(0.105)	(0.209)	(0.236)
Observations	31	31	31
R-squared	0.017	0.374	0.810
F-statistic	0.274	8.542	45.89

Newey-West standard errors in parentheses for Panels A-D and robust standard errors in parentheses for Panels E-H

*** p<0.01, ** p<0.05, * p<0.1

Appendix Table A1: Variable Definitions

Variable Name	Definition
Bitcoin Price	PX_LAST of XBT Index from Bloomberg showing Bitcoin price in USD
Bitcoin Return	Daily percentage change of the Bitcoin Price Index (XBT Index from Bloomberg)
Fund Actual Total Return	It is calculated as the percentage change in the Fund's market price and adjusted for all cash distributions, including dividends, interest, and realized capital gains. This measure is also referred to as the Fund's Market Price Return, Total Return, or Actual Return.
Fund Assets Under Management	Total Assets under Management (AUM) of the Fund in million USD
Fund Bid-Ask Spread	Daily bid-ask spread as a percentage of the mid price of the Fund; calculated as $[100 \times 2 \times (\text{ask price} - \text{bid price}) / (\text{ask price} + \text{bid price})]$
Fund Compounding Deviation	The Compounding Deviation is defined only for multi-day holding periods of leveraged ETFs. It is calculated as the difference between the Fund Target Return and the Fund naïve Expected Return, and is also referred to as the Compounding Effect.
Fund Flow	Daily percentage inflow (outflow) of capital into (out of) an ETF
Fund Inefficiency Return Deviation	Actual Total Return of the Fund minus its NAV return
Fund Naïve Expected Return	The Fund's naïve Expected Return is defined only for multiple holding days. It equals the cumulative underlying index return over the holding period multiplied by the fund's leveraged multiple. This naïve
Fund NAV	Net Asset Value (FUND_NET_ASSET_VAL from Bloomberg) of the Fund
Fund NAV Return	Return of a Fund based on the percentage change in NAV, adjusted for dividend yield.
Fund NAV Return Deviation	Fund NAV Return minus Fund Target Return
Fund Premium	The Fund's market valuation premium. Computed as the percentage deviation of the fund's market price from its NAV. A negative market valuation premium indicates a market valuation discount.
Fund Price	Daily closing price (PX_LAST from Bloomberg) of the Fund
Fund Target Return	For each trading day, the Fund's Target Return equals the fund's leveraged multiple times the daily underlying index return. Over multiple holding days, the Fund's Target Return reflects the compounded daily target returns across the holding period.
Fund Total Return Deviation	It is the Actual Total Return of an LETF minus the daily leveraged multiple times the return on the underlying index. On a daily basis, it can be decomposed into NAV Deviation and Inefficiency Deviation. For a holding period longer than one trading day, it can be decomposed into Compounding Deviation, NAV Deviation, and Inefficiency Deviation.
Fund Turnover	Calculated as the daily trading volume divided by the number of shares outstanding of the Fund.
Fund Underlying Index Return	The Fund's underlying index return (in %), which corresponds to the Bitcoin index return for Bitcoin ETFs and the Gold index return for Gold ETFs.
Holding-period Federal Funds Interest	The Federal Funds Interest accumulated over the holding period. In particular, the Fed Funds Interest earned over 1 trading day is calculated using the actual number of calendar days between the previous trading day and the current trading day, divided by 360, and then multiplied by the Fed Funds rate from the previous trading day. For multiple holding days, this measure is accumulated over each day during the holding period.
LETF	Leveraged exchange-traded funds (LETFs) are typically defined as ETFs that track the return on an underlying index (such as Bitcoin Return or Gold Return in this study) with a daily target multiple such as (2x), (-1x), and (-2x).
Multiple (m) of the LETF	Also called leveraged multiple, target multiple, or daily target multiple. Reflects the LETF's daily target exposure to the underlying index. The multiple can be positive (for bull LETFs) or negative (for bear/inverse LETFs). Examples of multiples for LETFs include (2x), (-1x), and (-2x), where m = 2, -1, and -2.
Multiple Function ($m^2 - m$)	Equals to the squared of the LETF multiple minus the multiple itself. It is equal to 2, 2, 6 for the (2x), (-1x), and (-2x) LETFs, respectively.
Gold Return	Daily percentage change of Gold Price in USD (XAU BGN Curncy from Bloomberg)
SPX Index return	Daily percentage change of the S&P 500 Total Return Index (SPXT Index from Bloomberg)

See Appendix Table A2 for the specifications of each Fund.

All return variables are in percentage, unless noted otherwise.

Appendix Table A2: List and Characteristics of U.S. Publicly-traded Bitcoin and Gold ETFs (as of December 2024)

ETF/ETN Type	Fund Ticker	Target Multiple	NAME	Inception Date	Premium/Discount (in %)	Asset under Management (Mil USD)	Market Capitalization (Mil USD)	Expense Ratio (%)	Average Daily Turnover (%)	Average Bid-Ask Spread (%)
Futures-based Bitcoin ETF	BITO	100%	PROSHARES BITCOIN ETF-USD	10/19/2021	0.00	2345.27	2345.20	0.95	9.70	0.04
Futures-based Bitcoin LETF	BITX	200%	VOLATILITY SHARES 2X BITCOIN STRATEGY ETF	6/27/2023	-0.52	3113.83	3097.76	1.90	14.76	0.09
Futures-based Bitcoin LETF	SBIT	-200%	PROSHARES ULTRASHORT BITCOIN	4/2/2024	0.18	54.58	54.68	0.95	29.52	0.09
Futures-based Bitcoin LETF	BITI	-100%	PROSHARES SHORT BITC ETF-USD	6/21/2022	0.11	98.96	99.07	1.03	39.89	0.04
Futures-based Bitcoin LETF	BITU	200%	PROSHARES ULTRA BITCOIN ETF	4/2/2024	-0.01	1094.79	1094.65	0.95	10.78	0.07
Spot-based GOLD ETF	GLD	100%	SPDR GOLD SHARES	11/18/2004	0.47	73215.07	73559.09	0.40	2.13	0.01
Futures-based GOLD LETF	UGL	200%	PROSHARES ULTRA GOLD	12/3/2008	0.03	289.69	289.79	0.95	6.44	0.11
Futures-based GOLD LETF	GLL	-200%	PROSHARES ULTRASHORT GOLD	12/3/2008	0.14	16.62	16.65	0.95	14.98	0.06
Futures-based GOLD LETN	DGZ	-100%	DB GOLD SHORT ETN	2/27/2008	.	.	2.64	0.75	0.24	8.22
Futures-based GOLD LETN	DZZ	-200%	DB GOLD DOUBLE SHORT ETN	2/27/2008	.	.	2.97	0.75	0.25	10.32

Source: Bloomberg; websites of the issuers of Exchange-Traded Funds (ETFs) and Exchange-Traded Notes (ETNs).